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序言：信息与行动的度量衡
——智能时代的语用信息论

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信息论正站在一个新的历史门槛上。70 多年来，香农理论回答了“符号如何被可靠传输”，控制论回答了“系统如何稳定运行”，决策论回答了“行动如何被理性选择”。这 3 条河流各自奔涌，深刻塑造了现代通信、计算与控制技术，却很少真正汇合。今天，当大语言模型、自动驾驶、具身智能和任务导向通信同时逼近一个共同问题——信息究竟如何转化为有效行动——我们比任何时候都更需要一种统一的数学语言。牛凯教授与张平院士的《语用信息的数学理论》，正是这样一部应运而生的著作。它不是对经典信息论的修补，而是把信息论从“符号世界”推进到“行动世界”的一次跃迁。换言之，它试图为智能时代建立一套“信息与行动的度量衡”。

这篇论文最核心的思想创新是提出“同效映射”，用以刻画“殊途同归”这一深刻原则：不同的语义路径，只要导向同一最优行动，就在语用层面等价。它与语义信息论中的“同义映射”构成自然层级：同义映射处理“同一意义可有多种表达”，同效映射则处理“同一行动可由多种意义支撑”。二者复合为“具象映射”，直接贯通语用目的与语法信号。由此，论文建立起语法、语义、语用三层级的信息框架。经典信息论与语义信息论不再是彼此分离的学说，而成为这一框架在映射退化为恒等时的特例。这个框架的优美之处在于，它没有否定香农，而是把香农安放在一个更宏大的坐标中：语法层保证信号可传，语义层保证意义可通，语用层保证行动有效。信息论由此从“描述世界”走向“指导行动”。

值得注意的是，这一思想触及了信息论中很少正面处理的一个根本问题：信息价值依赖目标。柯尔莫哥洛夫复杂度关注最短描述，香农熵关注概率不确定性，而语用信息关注对行动的影响。同一对象对不同任务具有不同描述长度和价值。本文用等价类商结构把这种目标相关性数学化，是对信息度量的一次重要扩展。它不再追问“信息是什么”，而是追问“信息为了什么”。这一转向，使信息论第一次能够以严格方式处理目的、效用与行动，为智能系统的信息处理提供了基础数学框架。

在基础理论层面，论文建立了一套完整的语用信息度量体系，并证明了 3 个基本编码定理：无损信源编码定理、信道编码定理与率失真编码定理。它们分别以语用熵、语用信道容量和语用率失真为极限，构成对香农三大定理在语用层面的推广。论文进一步引入语用信息价值与语用信息成本，作为率失真与信道容量的对偶，通过拉格朗日框架把功率、带宽、时延等物理资源换算为效用单位，并得到“行为容量”这一核心概念：资源受限智能体通过行动从环境中提取的最大净效用。如果说香农容量刻画了物理信道的极限，那么行为容量刻画了智能体在资源约束下通过行动“有所作为”的极限。这是语用信息框架下通信、控制与决策的首次统一，其理论意义远超单一学科。

论文还进一步扩展到连续消息与动态场景。对

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于高斯信源通过高斯信道的情况，论文给出闭式表达，说明语用等价类能够带来“等效信噪比放大”，从而显著降低达到目标任务所需的功率。对于动态决策，论文用贝尔曼方程刻画信念状态下的联合优化，把通信速率视为资源，把信念更新视为决策依据，形成感知–通信–决策–执行闭环的严格模型。这使理论不仅适用于静态编码，也适用于自动驾驶、具身智能、网络控制系统等序列决策场景。尤其可贵的是，论文始终没有停留在抽象定义，而是努力给出可计算、可优化、可验证的结构，这使它在工程上具有真实的落地潜力。

从多学科交叉融合的视角看，这篇论文的意义更为深远。它天然连接通信、控制、决策、人工智能、认知科学。在通信领域，它为任务导向通信和 6G 智能网络提供了理论基础；在人工智能领域，它为具身智能、智能体决策和人机协作提供了统一语言；在认知科学领域，它回应了“信息如何影响行动”这一根本问题。长期以来，这些学科各自发展出关于信息、意义和行动的理论，却缺乏共同的数学基础。本文所建立的语用信息框架，正是朝这一方向迈出的关键一步。它让我们看到，信息论不仅可以描述信道，也可以描述智能；不仅可以计算比特，也可以计算行动的价值。这正是智能时代构建“信息与行动度量衡”的努力方向。

当然，这一理论仍需继续发展。非平稳过程、有限块长、多智能体博弈、高维信念计算，以及由深度学习得到的同效映射的误差与泛化，都是未来值得研究的方向。但方向已经清楚：未来智能系统不应只是符号管道，而应是有目标、有资源约束、能从信息中提取行动价值的智能体。论文为这样的系统提供了严格而优美的数学起点。它不仅是一部理论著作，更是一种新的信息观：信息的终极意义，不在于被传输，而在于指导行动。

我期待这篇论文在未来 10 年引发更多跨学科研究，并成为从香农信息论走向智能信息论的重要里程碑。谨以此为序。

Preface: the measure of information and action pragmatic—information theory in the age of intelligence

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Information theory is standing at a new historical threshold. For more than seventy years, Shannon theory has answered "how symbols can be reliably transmitted," cybernetics has answered "how systems can operate stably," and decision theory has answered "how actions can be rationally chosen." These three rivers have flowed separately, deeply shaping modern communication, computing, and control technologies, yet they have rarely truly converged. Today, as large language models, autonomous driving, embodied intelligence, and task-oriented communication all approach a common question, how information can be transformed into effective action, we need a unified mathematical language more than ever. The Mathematical Theory of Pragmatic Information by Professor Niu Kai and Academician Zhang Ping is precisely a work that meets this need. It is not a revision of classical information theory, but a leap that advances information theory from the world of symbols to the world of action. In other words, it seeks to establish a measure of information and action for the age of intelligence.

The most central intellectual innovation of this work is the proposal of the isoteleia mapping, which characterizes the profound principle of equifinality: different semantic paths, as long as they lead to the same optimal action, are equivalent at the pragmatic level. Together with the synonymous mapping in semantic information theory, it forms a natural hierarchy: the synonymous mapping deals with the fact that the same meaning can have multiple expressions, while the isoteleia mapping deals with the fact that the same action can be supported by multiple meanings. The two combine into the reification mapping, which directly connects pragmatic purpose with syntactic signals. In this way, the work establishes a three-tier information framework of syntax, semantics, and pragmatics. Classical information theory and semantic information theory are no longer separate theories; they become special cases of this framework when the mappings degenerate into identity. The beauty of this framework is that it does not reject Shannon. Instead, it places Shannon within a broader coordinate system: the syntactic level ensures that signals can be transmitted, the semantic level ensures that meaning can be communicated, and the pragmatic level ensures that action can be effective. Information theory thus moves from describing the world to guiding action.

More notably, this idea touches a fundamental question that has long been avoided in information science: the value of information depends on the goal. Kolmogorov complexity focuses on the shortest description, Shannon entropy focuses on probabilistic uncertainty, and pragmatic information focuses on the impact on action. The same object has different de-

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scription lengths and values for different tasks. This work uses a quotient structure of equivalence classes to express this goal dependence in mathematical terms, which is an important extension of information measurement. It no longer asks "what information is," but asks "what information is for." This shift enables information theory, for the first time, to deal rigorously with purpose, utility, and action, and provides a foundational mathematical framework for information processing in intelligent systems.

At the level of basic theory, the work establishes a complete system of pragmatic information measures and proves three basic coding theorems: the lossless source coding theorem, the channel coding theorem, and the rate-distortion coding theorem. Their limits are respectively pragmatic entropy, pragmatic channel capacity, and pragmatic rate-distortion, and together they constitute a generalization of Shannon's three theorems at the pragmatic level. The work further introduces pragmatic value of information and pragmatic cost of information as duals of rate-distortion and channel capacity. Through a Lagrangian framework, it converts physical resources such as power, bandwidth, and delay into utility units, and obtains the core concept of behavioral capacity: the maximum net utility that a resource-constrained agent can extract from its environment through action. If Shannon capacity characterizes the limit of a physical channel, then behavioral capacity characterizes the limit of what an agent can achieve through action under resource constraints. This is the first time that communication, control, and decision have been brought into the same optimization framework under the pragmatic information framework, and its theoretical significance goes far beyond any single discipline.

The work further extends to continuous messages and dynamic scenarios. For a Gaussian source over a Gaussian channel, the work provides a closed-form expression showing that pragmatic equivalence classes can produce an effective signal-to-noise ratio gain, thereby significantly reducing the power needed to achieve the target task. For dynamic decision-making, the work uses the Bellman equation to characterize joint optimization under belief states, treating communication rate as a resource and belief update as the basis for decision-making, and forms a rigorous model of a perception-communication-decision-execution loop. This makes the theory applicable not only to static coding but also to sequential decision-making scenarios such as autonomous driving, embodied intelligence, and networked control systems. It is especially valuable that the work never remains at abstract definitions but strives to provide structures that are computable, optimizable, and verifiable, which gives it real potential for engineering implementation.

From the perspective of multidisciplinary integration, the significance of this work is even more profound. It naturally connects communication, control, decision, artificial intelligence, and cognitive science. In communication, it provides a theoretical foundation for task-oriented communication and 6G intelligent networks. In artificial intelligence, it provides a unified language for embodied intelligence, agent decision-making, and human-machine collaboration. In cognitive science, it responds to the fundamental question of how information affects action. For a long time, these disciplines have each developed their own theories about information, meaning, and action, yet they have lacked a common mathematical foundation. The pragmatic information framework established in this work is a key step in this direction. It shows us that information theory can describe not only channels but also intelligence, and can calculate not only bits but also the value of action. This is precisely the measure of information and action for the age of intelligence.

Of course, this theory still needs further development. Non-stationary processes, finite blocklength, multi-agent games, high-dimensional belief computa-

tion, and the errors and generalization of isoteleia mappings learned by deep learning are all directions worthy of future research. But the direction is clear: future intelligent systems should not be mere symbol pipelines, but goal-oriented, resource-constrained agents that can extract action value from information. This work provides a rigorous and elegant mathematical starting point for such systems. It is not only a theoretical work but also a new view of information: the ultimate meaning of information lies not in being transmitted but in guiding action.

I look forward to this work inspiring more interdisciplinary research in the next decade and becoming an important milestone in the journey from classic information theory to intelligent information theory. With these words, I write this preface.

ABOUT THE AUTHOR



Professor Li Ming is a Fellow of The Royal Society of Canada. He earned his Ph.D. from Cornell University in the United States and completed his postdoctoral research at Harvard University. He has held teaching positions at Ohio State University, Harvard University, York University, the University of Waterloo, and the University of California, Santa Barbara. In 2024, he received the IEEE Computer Society Wallace McDowell Award, and in 2010, he was awarded the Killam Prize, Canada's national science award. He is a Fellow of both the ACM and the IEEE, as well as a recipient of the Killam Fellowship and the E.W.R. Steacie Fellowship. Professor Li is a world-renowned authority on Kolmogorov complexity and an expert in bioinformatics.